

Name: Solutions

1. Determine whether the function $f(x) = x^2 + 3x - 10$ satisfies the hypotheses of the Mean Value Theorem on the interval $[0, 4]$. If it does, then find the value of c that

satisfies the equation $f'(c) = \frac{f(b) - f(a)}{b - a}$.

$f(x)$ is continuous on $[0, 4]$ and differentiable on $(0, 4)$
secant slope $m = \frac{f(4) - f(0)}{4 - 0} = \frac{28}{4} = 7$
 $f'(x) = 2x + 3$ $f'(c) = 2c + 3$ $2c + 3 = 7$ $c = 2$

2. Determine whether the function $f(x) = \cos x$ satisfies the hypotheses of the Mean Value Theorem on the interval $[0, 2\pi]$. If it does, then find the value of c that satisfies

the equation $f'(c) = \frac{f(b) - f(a)}{b - a}$.

$f(x)$ is continuous on $[0, 2\pi]$ and differentiable on $(0, 2\pi)$
secant slope $m = \frac{f(2\pi) - f(0)}{2\pi - 0} = \frac{1 - 1}{2\pi} = \frac{0}{2\pi} = 0$
 $f'(x) = -\sin x$ $f'(c) = -\sin c$ $0 = -\sin c$ $\sin c = 0$
 $c = \pi$

3. Determine whether the function $f(x) = \frac{x}{x-1}$ satisfies the hypotheses of the Mean Value Theorem on the interval $[0, 2]$. If it does, then find the value of c that satisfies the

equation $f'(c) = \frac{f(b) - f(a)}{b - a}$.

$f(x)$ is not continuous on $[0, 2]$ since $f(1) = \text{undefined}$

4. Determine whether the function $f(x) = x^{\frac{4}{3}}$ satisfies the hypotheses of the Mean Value Theorem on the interval $[-1, 8]$. If it does, then find the value of c that satisfies the

equation $f'(c) = \frac{f(b) - f(a)}{b - a}$.

$f(x)$ is continuous on $[-1, 8]$ $f'(x) = \frac{4}{3} x^{\frac{1}{3}}$

$f(x)$ is differentiable on $(-1, 8)$

secant slope $m = \frac{16 - 1}{8 - (-1)} = \frac{15}{9} = \frac{5}{3}$ $f'(c) = \frac{4}{3} c^{\frac{1}{3}}$

$\frac{5}{3} = \frac{4}{3} c^{\frac{1}{3}}$ $\frac{5}{4} = c^{\frac{1}{3}}$ $c = \frac{125}{64}$

5. Determine whether the function $f(x) = x^{\frac{2}{3}}$ satisfies the hypotheses of the Mean Value Theorem on the interval $[-1, 27]$. If it does, then find the value of c that satisfies the

equation $f'(c) = \frac{f(b) - f(a)}{b - a}$.

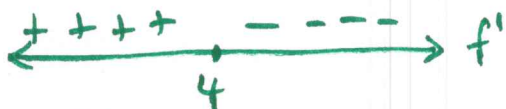
$f(x)$ is continuous on $[-1, 27]$ $f'(x) = \frac{2}{3} x^{-\frac{1}{3}}$

$f'(x) = \frac{2}{3 x^{\frac{1}{3}}}$

$f(x)$ is not differentiable at $x=0$

6. Determine the interval(s) on which the function $f(x) = 8x - x^2$ is (a) increasing and (b) decreasing.

$f'(x) = 8 - 2x$ $0 = 8 - 2x$ $x = 4$



$f(x)$ is increasing on $(-\infty, 4)$ since $f'(x) > 0$

$f(x)$ is decreasing on $(4, \infty)$ since $f'(x) < 0$

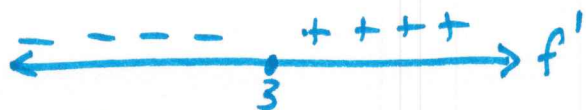
7. Determine the interval(s) on which the function $f(x) = e^x$ is (a) increasing and (b) decreasing.

$f'(x) = e^x$ $e^x \neq 0$ $e^x > 0$ on $(-\infty, \infty)$

$f(x)$ is increasing on $(-\infty, \infty)$

8. Determine the interval(s) on which the function $f(x) = e^{x^2-6x}$ is (a) increasing and (b) decreasing.

$$f'(x) = (2x-6)e^{x^2-6x} \quad f'(3) = 0$$

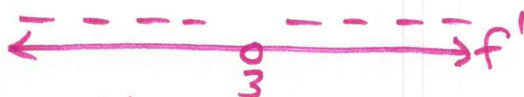


$f(x)$ is increasing on $(3, \infty)$ since $f'(x) > 0$

$f(x)$ is decreasing on $(-\infty, 3)$ since $f'(x) < 0$

9. Determine the interval(s) on which the function $f(x) = \frac{1}{x-3}$ is (a) increasing and (b) decreasing.

$$f'(x) = \frac{-1}{(x-3)^2} \quad \frac{-1}{(x-3)^2} \neq 0 \quad f'(3) = \text{undefined}$$



$f(x)$ is never increasing.

$f(x)$ is decreasing on $(-\infty, 3) \cup (3, \infty)$ since $f'(x) < 0$

10. Determine the interval(s) on which the function $f(x) = \frac{x^3}{3} + \frac{x^2}{2} - 6x + 5$ is (a)

increasing and (b) decreasing.

$$f'(x) = x^2 + x - 6 \quad f'(x) = 0$$

$$0 = x^2 + x - 6 \quad 0 = (x+3)(x-2) \quad f'(-3) = 0 \quad f'(2) = 0$$



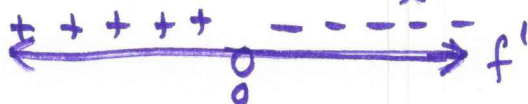
$f(x)$ is increasing on $(-\infty, -3) \cup (2, \infty)$ since $f'(x) > 0$

$f(x)$ is decreasing on $(-3, 2)$ since $f'(x) < 0$

11. Determine the interval(s) on which the function $f(x) = \frac{1}{x^2}$ is (a) increasing and (b)

decreasing.

$$f'(x) = -\frac{2}{x^3} \quad f'(x) \neq 0 \quad f'(0) = \text{undefined}$$



$f(x)$ is increasing on $(-\infty, 0)$ since $f'(x) > 0$

$f(x)$ is decreasing on $(0, \infty)$ since $f'(x) < 0$

12. Use the Concavity Test to determine the interval(s) on which the graph of the function

$f(x) = x^3 - 3x^2 - 9x + 1$ is (a) concave up and (b) concave down.

$$f'(x) = 3x^2 - 6x - 9 \quad f''(x) = 6x - 6 \quad f''(1) = 0$$



$f(x)$ is concave up on $(1, \infty)$ since $f''(x) > 0$

$f(x)$ is concave down on $(-\infty, 1)$ since $f''(x) < 0$

13. Use the Concavity Test to determine the interval(s) on which the graph of the function

$f(x) = x^4 - 12x^3 + 48x^2 - x + 2$ is (a) concave up and (b) concave down.

$$f'(x) = 4x^3 - 36x^2 + 96x - 1 \quad f''(x) = 12x^2 - 72x + 96$$

$$0 = 12x^2 - 72x + 96 \quad 0 = x^2 - 6x + 8 \quad 0 = (x-2)(x-4)$$

$$f''(2) = 0 \quad f''(4) = 0$$



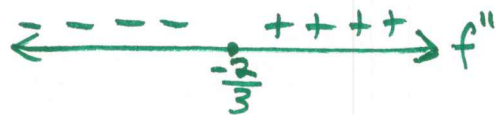
$f(x)$ is concave up on $(-\infty, 2) \cup (4, \infty)$ since $f''(x) > 0$

$f(x)$ is concave down on $(2, 4)$ since $f''(x) < 0$

14. Find all the points of inflection of the function $f(x) = x^3 + 2x^2 - x - 2$

$$f'(x) = 3x^2 + 4x - 1 \quad f''(x) = 6x + 4 \quad f''(x) = 0$$

$$0 = 6x + 4 \quad 6x = -4 \quad x = -\frac{2}{3}$$

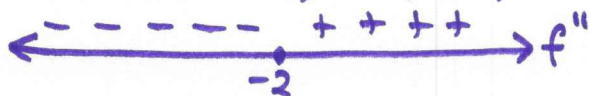


$f(x)$ has a point of inflection at $x = -\frac{2}{3}$ since $f(x)$ has a tangent line at $x = -\frac{2}{3}$ and $f''(x)$ changes sign at $x = -\frac{2}{3}$

15. Find all the points of inflection of the function $f(x) = xe^x$

$$f'(x) = e^x + xe^x \quad f''(x) = e^x + e^x + xe^x = 2e^x + xe^x$$

$$0 = e^x(2+x) \quad f''(-2) = 0$$



$f(x)$ has a point of inflection at $x = -2$ since $f(x)$ has a tangent at $x = -2$ and $f''(x)$ changes sign at $x = -2$

16. Find all the points of inflection of the function $f(x) = x^4 - 6x^2 + 12$

$$f'(x) = 4x^3 - 12x \quad f''(x) = 12x^2 - 12 \quad 0 = 12x^2 - 12$$

$$0 = x^2 - 1 \quad f''(-1) = 0 \quad \text{and} \quad f''(1) = 0$$



$f(x)$ has points of inflection at $x = -1$ and $x = 1$ since $f(x)$ has tangents at $x = -1$ and $x = 1$ and since $f''(x)$ changes sign at $x = -1$ and $x = 1$